VII-5. PERTURBATION OF OPTICAL RESONATOR CHARACTERISTICS BY AN INHOMOGENEOUS FOCUSING MEDIUM

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The perturbation produced in the stability characteristics of an empty curved mirror optical resonator upon insertion of an inhomogeneous focusing medium has been examined in the general case wherein the space between mirrors is only partially occupied by such a medium. Stability of optical modes in a resonator is closely related to the behavior of paraxial rays. Such rays are either confined to the structure (termed "stable") or diverge from the structure ("unstable"). In the formulation presented here the significant parameters result directly from the multiplication of transfer matrices in a somewhat different form than that previously given by Kogelnik.[1] The importance of the configuration analyzed is underlined by the experimental observations of Welling[2] and others regarding the presence of refractive index gradients in flashed laser materials. The computed results suggest an experiment employing the losses of a resonator operating near the instability boundary as an independent indication of the refractive gradient.

First order ray theory considers the passage of paraxial rays through systems of large aperture. Algebraization of such rays is accomplished by assigning to each ray at planes transverse to the axis of the optical system the pair of parameters a_N and b_N. These indicate the height above the optic axis and slope of the ray, respectively, at the Nth reference plane. The transformations of these parameters through elementary optical system components are expressed in the form of transfer matrices analogous to those found in electric filter theory[3]. Useful equivalent microwave circuits are readily established and involve both reciprocal and nonreciprocal elements[4]. Ray matrices have been developed[5] for a length of homogeneous medium, lens, mirror, and dielectric interfaces. These matrices together with resistance and reactance equivalents are presented in Table I.

Consider the resonator (Fig. 1a) composed of mirrors of arbitrary curvatures separated by lengths ℓ_1 and ℓ_2 of homogeneous media together with length ℓ_3 of a longitudinally uniform focusing medium characterized by an index of refraction $n(y) = n_0 - n_2 y^2$. From the equivalent lens sequence for the resonator we readily establish an equivalent microwave network and extract a symmetric unit cell (Fig. 1b). The nonreciprocal elements employed are so disposed in the circuit as to render the overall unit cell reciprocal. The transfer matrix for a unit cell is:

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$$T = \begin{pmatrix} 1 & 0 \\ -K_1 & 1 \end{pmatrix} \begin{pmatrix} 1 & \ell_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{n_0} \end{pmatrix} \begin{pmatrix} \cos\beta\ell_3 & Z\sin\beta\ell_3 \\ -Y\sin\beta\ell_3 & \cos\beta\ell_3 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & n_0 \end{pmatrix}$$

$$\cdot \begin{pmatrix} 1 & \ell_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -2K_2 & 1 \end{pmatrix} \begin{pmatrix} 1 & \ell_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & n_0 \end{pmatrix} \begin{pmatrix} \cos\beta\ell_3 & Z\sin\beta\ell_3 \\ -Y\sin\beta\ell_3 & \cos\beta\ell_3 \end{pmatrix}$$

$$\cdot \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{n_0} \end{pmatrix} \begin{pmatrix} 1 & \ell_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -K_1 & 1 \end{pmatrix} = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix} \tag{1}$$

$$\begin{aligned} t_{11} = {} & \cos^2\beta\ell_3 \left[1 - 2K_1\left(\ell_1+\ell_2\right) - 2K_2\left(\ell_1+\ell_2-\ell_1\ell_2\right) + 2K_1K_2\left(\ell_1^2+\ell_2^2+\ell_1\ell_2\right)\right] \\ & + \sin^2\beta\ell_3 \left[\frac{\ell_1K_1}{n_0^2} - \frac{2\ell_1\ell_2K_1K_2}{n_0^2} + \frac{2K_1K_2}{\beta^2} + \left(1-\ell_1K_1\right)\right. \\ & \left.\left(2\ell_1\ell_2\beta^2 - 2\ell_1\ell_2^2K_2\beta^2 - n_0^2 + 2\ell_2K_2n_0^2\right)\right] \\ & + \sin\beta\ell_3\cos\beta\ell_3 \left[\left(1-\ell_1K_1\right)\left(\frac{2\ell_1\ell_2K_2\beta}{n_0} - \frac{\ell_1\beta}{n_0} - \frac{2K_2n_0}{\beta}\right.\right. \\ & \left. - 2\ell_2n_0\beta + 2\ell_2^2K_2n_0\beta - \ell_1n_0\beta + 2\ell_1\ell_2K_2n_0\beta\right) \\ & + \frac{1}{\beta n_0}\left(2\ell_2K_1K_2-K_1+2\ell_1K_1K_2\right) + \frac{1}{\beta}\left(2\ell_2K_1K_2n_0-K_1n_0\right) \\ & \left. + \frac{1}{n_0}\left(2\ell_1\ell_2K_1\beta-2\ell_1\ell_2^2K_1K_2\beta\right)\right] \end{aligned} \tag{2}$$

The stability condition for the resonator is readily expressed in terms of the focal lengths, spacings, and indices of refraction; Pierce's criterion has been shown equivalent to[1,5]

$$\left|\frac{1}{2}\text{ trace } T\right| \leq 1 \tag{3}$$

where

$$\frac{1}{2}\text{ trace } T = \frac{1}{2}\left(t_{11}+t_{22}\right) = t_{11} \tag{4}$$

Specializing to the case of a resonator completely filled by an inhomogeneous medium by letting ℓ_1 and ℓ_2 go to zero in eq. (2), we obtain

$$t_{11} = \cos^2\beta\ell + \sin^2\beta\ell\left(\frac{2K_1K_2}{\beta^2} - 1\right) - \frac{2}{\beta}\sin\beta\ell\cos\beta\ell\left(K_1+K_2\right) \tag{5}$$

where $\ell = \ell_3$ = separation between mirrors.

Stability plots (Fig. 2) for initially unstable resonators of mirror separation $\ell = 1$ are presented. A point on the axis, $\beta\ell = 0$, corresponds to an infinitely degenerate set of resonators without the presence of focusing effect in the medium. As the medium acquires optical focusing properties, $\beta\ell \neq 0$, the degeneracy is lifted, and trajectories corresponding to typical individual resonator configurations are shown. The nature of the splitting produced by the focusing effect may be understood qualitatively as follows. Consider an initially unstable resonator, Kahn[5] has shown that the virtual centers of curvature for Siegman's[6] spherical waves, are related to the resonator characteristic impedances $(jZ)^2 = (\overline{Z})^2 = t_{12}/t_{21}$. Virtual centers positioned outside the resonator indicate a divergent outgoing spherical wave upon reflection. A significant feature of the curves therefore is that in such instances the half-trace, for increasing convergence of the medium (β), tends toward more stable regions as the physical argument indicates.

The loss due to the finite sizes of curved mirrors when these form an unstable optical resonator has been related explicitly to the half-trace:

$$\frac{1}{2}\tau = \cosh \psi , \qquad (6)$$

where ψ is the fractional loss per pass in nepers.

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OPTICAL	RESISTANCE ANALOG		REACTANCE ANALOG	
	TRANSFER MATRIX	CIRCUIT	TRANSFER MATRIX	CIRCUIT
LENGTH ℓ OF HOMOGENEOUS MEDIUM	$\begin{pmatrix} 1 & \ell \\ 0 & 1 \end{pmatrix}$	$-\ell$ RESISTANCE $= -\ell$	$\begin{pmatrix} 1 & -j\ell \\ 0 & 1 \end{pmatrix}$	ℓ REACTANCE $= \ell$
SIMPLE THIN LENS $\left(K = \frac{1}{f}\right)$	$\begin{pmatrix} 1 & 0 \\ -K & 1 \end{pmatrix}$	K CONDUCTANCE = K	$\begin{pmatrix} 1 & 0 \\ -jK & 1 \end{pmatrix}$	K SUSCEPTANCE = K
DIELECTRIC INTERFACES (INDICES n_1 AND n_2)	$\begin{pmatrix} 1 & 0 \\ 0 & n_1/n_2 \end{pmatrix} = \sqrt{\frac{n_1}{n_2}} \begin{pmatrix} \sqrt{\frac{n_2}{n_1}} & 0 \\ 0 & \sqrt{\frac{n_1}{n_2}} \end{pmatrix}$	$1:\sqrt{n_2/n_1}$ $\sqrt{\frac{n_1}{n_2}}$ WHEELER RATIO REPEATER	(SAME)	
LENGTH ℓ OF INHOMOGENEOUS FOCUSING MEDIUM $n(Y) = n_0 - n_2 Y^2$ $\beta = \sqrt{\frac{2n_2}{n_0}}$	$\begin{pmatrix} \cos\beta\ell & Z\sin\beta\ell \\ -Y\sin\beta\ell & \cos\beta\ell \end{pmatrix}$		$\begin{pmatrix} \cos\beta\ell & -jZ\sin\beta\ell \\ -jY\sin\beta\ell & \cos\beta\ell \end{pmatrix}$	$Y = \beta$
THIN PRISM	α, n_1 n_2 n_1	B, $B = \left(\frac{n_2}{n_1} - 1\right)\alpha$		
FLAT PLATE	Φ ℓ D AXIS $(\Phi \ll 1)$ n_1 n_2 n_1	$-(n_1/n_2)D$, A, $A = -D\Phi\left(1 - \frac{n_1}{n_2}\right)$		

TABLE I: ELEMENTARY OPTICAL COMPONENTS, THEIR TRANSFER MATRICES AND ELECTRIC ANALOGS

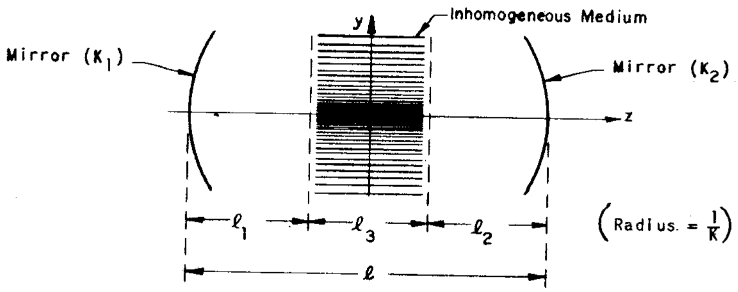

Figure 1a. Resonator Incompletely Filled with Inhomogeneous Focusing Medium

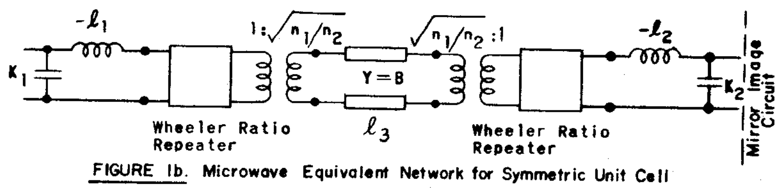

FIGURE 1b. Microwave Equivalent Network for Symmetric Unit Cell

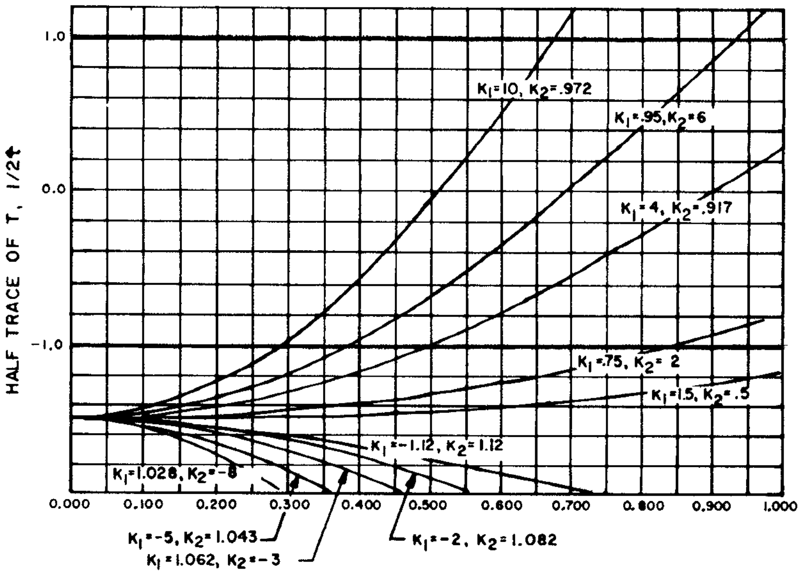

FIGURE 2. STABILITY PLOT FOR INITIALLY UNSTABLE RESONATOR